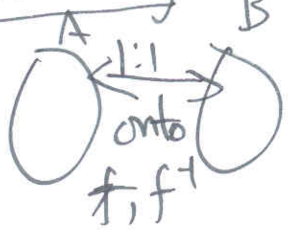
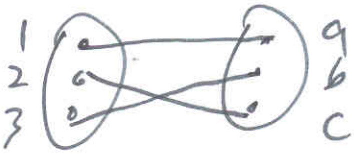
3/16. Uncountable & algorithms

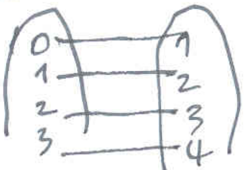
Cardinality



$|A| = |B|$

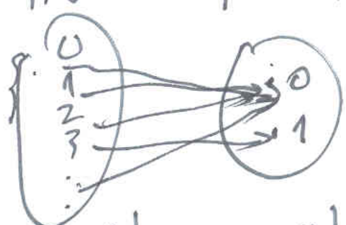


infinite set



Cantor's Diagonal argument

$f: \mathbb{N} \to \{0, 1\}$



(0 1 0 . . . .) infinite binary string
position 0 1 2 3

$\updownarrow$

$\{1, \dots \} \subseteq \mathbb{N}$

$\{0,1\}^{\mathbb{N}} \cong 2^{\mathbb{N}}$

$\downarrow$ infinite binary string

$\downarrow$ powerset of $\mathbb{N}$

$A_0 = (a_{00}, a_{01}, a_{02}, \dots)$ $\forall i,j \in \mathbb{N}\ a_{ij} \in \{0, 1\}$

$A_1 = (a_{10}, a_{11}, a_{12}, \dots$

$A_2 = (a_{20}, a_{21}, a_{22}, \dots)$

$\vdots$

$A_i = (a_{i0}, a_{i1}, a_{i2}, \dots a_{ii} \dots)$

$B = (b_0, b_1, b_2, \dots, b_i, \dots)$

$b_i = 0$ if $a_{ii} = 1$

$= 1$ if $a_{ii} = 0$ (otherwise)

$\mathbb{N} \times \mathbb{N}$ ... countable $\mathbb{N}^2$ .. $\cancel{\mathbb{Q}^2}$ $\mathbb{N}^k$ .. countable

$V^* = \bigcup_{i \in \mathbb{N}} V^i = V^0 \cup \underline{V^1} \cup V^2 \cup \dots$ .. countable

$(\varepsilon), (a, b, \dots z), (aa, \quad , zz)$

2) Countable 증명하기 (proof)

Numbering

$E \underset{\frac{1}{2}n \rightarrow \quad \leftarrow 2\cdot n}{\cong} \mathbb{N}$

$\mathbb{Z} \cong \mathbb{N}$? ? $\qquad \mathbb{Q} \cong \mathbb{N}$? ?

$V^* \cong \mathbb{N}$? ?

countable < finite / infinite — countably infinite

uncountable (infinite)

Chap 3 Algorithms, Integers,

3.1 Algorithms

Dijkstra's mini language

$x := x+5;\quad x, y := y, x$

concurrent assignment

$t := x;\ x := y;\ y := t$

if B_1 then if B_2 then S_A else S_B fi

fi fi

dangling else ambiguity

if $B_1 \rightarrow SL_A$
[] $\neg B_1 \rightarrow SL_A$
fi

if $B_1 \rightarrow SL_1$
| $B_2 \rightarrow SL_2$
:
| $B_n \rightarrow SL_n$
fi