1) 3/9 Chap 2. Sets, Functions, Sequences, and Sum 1

2.1 Sets

- A set is an unordered collection of objects.
- Cantor's naive set theory (Cantor's diagonal arg.)

Russel's paradox

$\boxed{S = \{x \mid x \notin x\}}$ — Denial of self recursion

$x \in S$ iff $x \notin S$

$\therefore S \in S$ iff $S \notin S$. X

若 $\begin{cases} x \in x \\ x \notin x \end{cases}$

$x = \{a, b, c, \{a, b, c\}\}$

Def. 2: $a \in A$, $a \notin A$
$\downarrow$
element

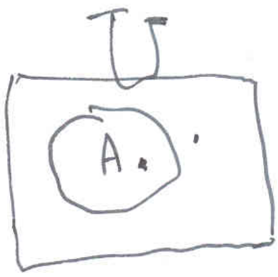
Two ways to define sets

i) $A = \{a_1, a_2, \cdots, a_n\}$ ... finite

$A = \{a_1, a_2, \cdots\}$ ... infinite

ii) $A = \{x \mid P(x)\}$
predicate

$A = \{x \in U \mid P(x)\}$
$\uparrow$
universe (type)



$\mathbb{N} = \{0, 1, 2, \cdots\}$

$\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$

$\mathbb{Z}^+ = \{1, 2, 3, \cdots\}$

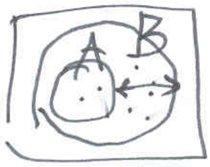
$\mathbb{Q} = \{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}$ and $q \neq 0\}$

$\mathbb{R} =$ set of real numbers

3) Def 4. $A \subseteq B$, iff every elements of A is also an element of B.

$$A \subseteq B \Leftrightarrow \forall x (x \in A \Rightarrow x \in B) \quad \left\langle \begin{array}{l} A \subsetneq B \\ A = B \end{array} \right.$$

$\equiv$



Def 3. $A = B$, iff $A \subseteq B \wedge B \subseteq A$.

$$A \subsetneq B \Leftrightarrow A \subseteq B \wedge (A \neq B)$$

$$\quad\quad\quad\quad\quad\quad\quad\quad (B \nsubseteq A)$$

Thm 1 $\emptyset \subseteq S$, $S \subseteq S$ $\quad \{\} = \emptyset \neq \{\emptyset\}$

Def.5. S $|S|$ … cardinality of S

size of S

Def 6. finite

infinite

Def 7. Set S,

$$P(S) = \{A \mid A \subseteq S\}$$

Ex) $S = \{1.2.3\}$

$$\mathcal{P}(S) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

000 001 010 100 011 101 110 111

$|S|$-bit binary string

$$|\mathcal{P}(S)| = 2^{|S|}$$

$$\mathcal{P}(S) = \underline{2^S}$$

3)

(3

ordered n-tuple

$(a_1, \cdots, a_n) = (b_1, \cdots, b_m)$

$n = m \wedge \forall i \in \{1 \ldots n\},\ a_i = b_i$.

$1 \leq {}^{\forall} i \leq n$.

$\forall i : 1 \leq i \leq n :$ 〃

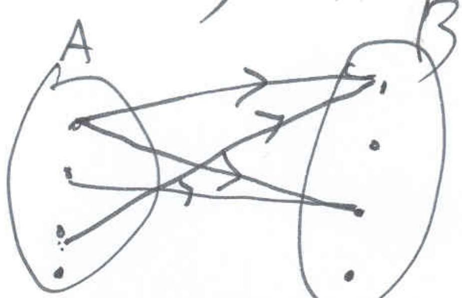
Cartesian product

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

$A \times B \neq B \times A$

$|A \times B| = |A| \times |B|$

If $R \subseteq A \times B$, R is called <u>relation</u> from A to B.



if $(a, b) \in R$, we write aRb.

Two interpretation of Rel

i) $R \subseteq A \times B$

ii) $R : A \times B \to \{t, f\}$

$= \subseteq \mathbb{N} \times \mathbb{N}$.

$<$

$\leq$

$\vdots$

$= \{(a, b) \mid a = b\}$

$= \{(0, 0), (1, 1), \cdots\}$

$= id_{\mathbb{N}}$

Def. $id_A = \{(a, a) \mid a \in A\}$

$id_A \subseteq A \times A$

$R \subseteq A \times A$: Relation on A.

graph $G = (V, E)$ (vertex, edge)

set $E \subseteq V \times V$