2007 Spring CS204 Homework #3

1. Given that A $\subseteq$ C and B $\subseteq$ D, show that A $\times$ B $\subseteq$ C $\times$ D.

Suppose $C = A \cup P$ and $D = B \cup Q$.

$C \times D \quad = (A \cup P) \times (B \cup Q)$

$\qquad = (A \times B) \cup (A \times Q) \cup (P \times B) \cup (P \times Q)$

$\therefore A \times B \subseteq C \times D$

2. Exercise 2.3.26

a) {1}

b) {-1, 1, 5, 9, 15}

c) {0, 1, 2}

d) {0, 1, 5, 16}

3. Exercise 2.3.28

a) {x| x is even.}

b) {x| x is even and x≥0}

c) R

4. Let f: $E \rightarrow F$ and g: $F \rightarrow G$ be two functions

(i) f and g injective $\Rightarrow$ $g \circ f$ injective.

(ii) f and g surjective $\Rightarrow$ $g \circ f$ surjective.

(iii) f and g bijective $\Rightarrow$ $g \circ f$ bijective.

Prove these properties.

(i) Let x, y in E be such that (g∘ f) (x) = (g∘ f)(y). Because g and f are injective we deduce that f(x) = f(y) and this implies that x = y. Hense, g ∘f is injective.

(ii) Let z in G Because g in surjective, there exists y in F such that g(y) = z. Because f is surjective, there exists x in E such that f(x)=y. We deduce that (g∘ f)(x) = z, which proves that g∘ f is surjective.

(iii) From (i) and (ii), it holds.

5. Show that the set of integers Z is countable.

There exists the bijection from Z to N such that

f(0) = 0

f(k) = 2k, if k is positive,

= -2k-1, if k is negative.

6. Show that the set of real numbers between 0 and 1 is uncountable.

0 과 1 사이의 실수를 다음과 같이 차례대로 자연수와 대응시킨다.

1 $0.a_{11}a_{12}a_{13}a_{14}\ldots$

2 $0.a_{21}a_{22}a_{23}a_{24}\ldots$

3 $0.a_{31}a_{32}a_{33}a_{34}\ldots$

4 $0.a_{41}a_{42}a_{43}a_{44}\ldots$

….

모두 대응시킨 다음에 위에서 나열되지 않은 0과 1사이의 실수가 존재함을 보인다.

$0.b_1b_2b_3b_4\ldots$, where $b_i = 1$ if $a_{ii} = 9$

$9 - a_{ii}$, if $a_{ii} = 0, 1, \ldots, 8$

로 잡으면 $0.b_1b_2b_3b_4\ldots$ 는 위에서 나열되지 않은 실수이다.

그러므로 0과 1사이의 실수는 uncountably infinite set이다.