2007 Spring CS204 Homework #2

1. Exercise 1.3.10

a) $\exists x(C(x) \wedge D(x) \wedge F(x))$

b) $\forall x(C(x) \vee D(x) \vee F(x))$

c) $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

d) $\neg\exists x(C(x) \wedge D(x) \wedge F(x))$

$\equiv \forall x \neg (C(x) \wedge D(x) \wedge F(x))$

$\equiv \forall x (\neg C(x) \vee \neg D(x) \vee \neg F(x))$

e) $\exists x\, C(x) \wedge \exists x\, D(x) \wedge \exists x\, F(x))$

2. Exercise 1.4.20

a) $\forall x \forall y(x<0 \wedge y < 0 \rightarrow xy > 0)$

b) $\forall x \forall y(x>0 \wedge y > 0 \rightarrow (x+y)/2 > 0)$

c) $\neg\forall x \forall y(x<0 \wedge y < 0 \rightarrow x - y < 0)$

d) $\forall x \forall y(\neg(|x + y| > |x| + |y|))$ or $\forall x \forall y(|x + y| \leq |x| + |y|)$

3. Exercise 1.4.28

a) T b) F c) T d) F e) T f) F g) T h) F i) F j) T

4. Exercise 1.5.28

(1) $\forall x(P(x) \vee Q(x))$	Premise
(2) $\forall x(P(x) \vee Q(x) \vee R(x))$	Addition from (1)
(3) $\forall x(\neg P(x) \wedge Q(x) \rightarrow R(x))$	Premise
(4) $\forall x(\neg(\neg P(x) \wedge Q(x)) \vee R(x))$	Logical equivalence ($p \rightarrow q \equiv \neg p \vee q$)
(5) $\forall x(P(x) \vee \neg Q(x) \vee R(x))$	De Morgan's Law
(6) $\forall x(P(x) \vee R(x))$	Resolution from (2) and (5)
(7) $\forall x(\neg R(x) \rightarrow P(x))$	Logical equivalence ($p \rightarrow q \equiv \neg p \vee q$)

5. Exercise 1.5.32

$(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$

$q = r = F$

$(p \vee F) \wedge (\neg p \vee F) \rightarrow (F \vee F)$ (resolution)

$\equiv p \wedge \neg p \rightarrow F$ (identity law, table 6, p. 24)

By modus ponens, $p \wedge \neg p \equiv F$.

6. Exercise 1.6.42

Every variables are integers.

(i) n^2 is odd. → (ii) $1 - n$ is even.

n^2 is odd → n is odd → $1 - n$ is even.

(ii) $1 - n$ is even. → (iii) n^3 is odd.

$1 - n = 2k$

$n = -2k + 1$

$n^3 = (-2k + 1)^3 = -8k^3 + 1 + 3(-2k)(-2k + 1)$

$= -8k^3 + 1 + 12k^2 - 6k$

$= 2(-4k^3 + 6k^2 - 3k) + 1$

Thus, n^3 is odd.

(iii) n^3 is odd. → (iv) $n^2 + 1$ is even.

n^3 is odd. → n is odd.

$n=2m + 1$

$n^2 + 1=(2m + 1)^2 + 1$

$= 4m^2 + 4m + 1 + 1$

$= 4m^2 + 4m + 2$

$= 2(2m^2 + 2m + 1)$

Thus, $n^2 + 1$ is even.

(iv) $n^2 + 1$ is even. → (i) n^2 is odd.

$n^2 + 1 = 2k$

$n^2 = 2k - 1 = 2(k - 1) + 1$

Thus, n^2 is odd.

7. Show that the modus ponens rule is equivalent to the modus tollens rule, i.e., we can prove the modus tollens rule from the modus ponens rule, and vice-versa.

We note that $p \rightarrow q$ is equivalent to $\neg q \rightarrow \neg p$.
Thus, $[\neg q \wedge (\neg q \rightarrow \neg p)] \rightarrow \neg p$ is equivalent to the modus tollens rule.
On the other hand, substituting $\neg q$ for p and $\neg p$ for q in the modus ponens rule, we have $[\neg q \wedge (\neg q \rightarrow \neg p)] \rightarrow \neg p$, which can thus be deduced from the modus ponens rule.

The converse can be proved similarly. We notice that $p \rightarrow q$ is equivalent to $\neg p \vee q$, then the modus ponens rule that is written $[p \wedge (p \rightarrow q)] \rightarrow q$ is equivalent to $p \rightarrow [(p \rightarrow q) \rightarrow q]$, and also to $p \rightarrow [\neg(p \rightarrow q) \vee q]$, whose contrapositive $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ is the modus tollens rule.