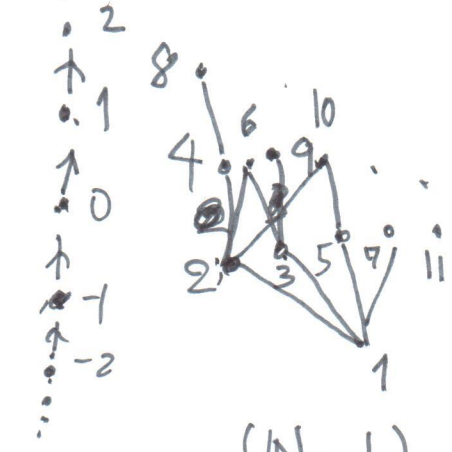
6/13(목) 개강 Boolean Algebra

Chap 3.

Let $(A, \leq)$ be a poset.

Ex)



$(\mathbb{Z}, \leq)$ $(\mathbb{N}_1, |)$

$a, b \in A$ $ub(a,b) = \{c \mid c \geq a, c \geq b\}$ 큰값

$lb(a,b) = \{c \mid c \leq a, c \leq b\}$ 작은값

$lub(a,b) = \max(a,b)$ $ub(a,b)$ = 배수

$glb(a,b) = \min(a,b)$ $lb(a,b)$ = 약수

lub

glb.

lub : 최소공배수

glb : 최대공약수

격자

$(A, \leq)$ poset is a lattice

$\forall a,b \in A$, $\exists$ lub(a,b), glb(a,b)

Ex) $A, B \in 2^S$ or $A, B \subseteq S$ where S is a set of all sets.

universe

Consider $(2^S, \subseteq)$

$ub(A,B) \stackrel{?}{=} \{C \subseteq S \mid A \subseteq C, B \subseteq C\}$

$lub(A,B) \stackrel{?}{=} A \cup B$

$lb(A,B) = \{C \mid C \subseteq A, C \subseteq B\}$

$glb(A,B) = A \cap B$

$(2^S, \cup, \cap)$

$(\mathbb{Z}, \leq) \longrightarrow (\mathbb{Z}, \max, \min)$ $\quad \leq \subseteq \mathbb{Z} \times \mathbb{Z} \longrightarrow \max, \min : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$

$(\mathbb{N}_1, |) \longrightarrow (\mathbb{Z}, LCM, GCD)$ $\quad | \subseteq \mathbb{N} \times \mathbb{N} \to LCM, GCD : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$(2^S, \subseteq) \longrightarrow (2^S, \cup, \cap)$ $\quad \subseteq \subseteq 2^S \times 2^S \to \cup, \cap : 2^S \times 2^S \to 2^S$

함수 uniqueness

3 Consider $(\{0,1\}, \leq)$ $\longrightarrow$ $(\{0,1\}, \vee, \wedge)$ $\leftrightarrow$ $(\{T,F\}, \vee, \wedge)$ $\leq$

poset

lattice

$\vee$	0	1
0	0	1
1	1	1

$\wedge$	0	1
0	0	0
1	0	1

$\vee$	F	T
F	F	T
T	T	T

$\wedge$	F	T
F	F	F
T	F	T

$(\{0,1\}, \geq)$ – principle of duality