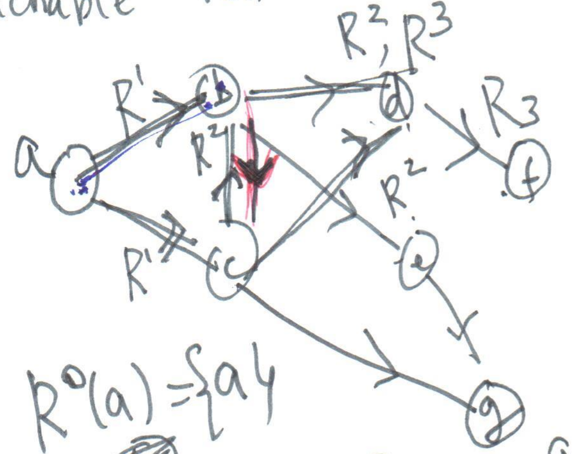
1)

5/18 강 Shortest Path

Reachable vertex in a directed graph $G=(V,E)$

$V \times V \subseteq E$



$R \subseteq A \times A$ $\quad R^n \subseteq A \times A$

R^n: $\begin{cases} R^0 =_B id_A & n=0 \\ R^n =_R R^{n-1} \circ R & n \geq 1 \end{cases}$

$R \circ T = \{\langle a,c\rangle \mid (a,b) \in R, (b,c) \in T\}$

$R^0(a) = \{a\}$

$a \in A$

$R^1(a) = \{b,c\}$

$R^2(a) = \{b,d,e\}$

$R^3(a) = \{d,e,f,g\}$

$R^3(a) =_R R^2 \circ R(a)$

$=_R R^1 \circ R \circ R(a)$

$=_R R^0 \circ R \circ R \circ R(a)$

$=_B id_A \circ R \circ R \circ R(a)$

$=_{\text{def. of } id_A} R \circ R \circ R(a)$

$R^0(a) = \{a\}$

$R^1(a) = \{b,c\}$

$R^2(a) = \{b,c,d,e,g$

Def $R^* = \bigcup_{i \in \mathbb{N}_0} R^i = R^0 \cup R^1 \cup R^2 \cup \ldots$ → reflexive & transitive closure of R

$R^0 = id_A$

$R^+ = \bigcup_{i \in \mathbb{N}_1} R^i = R^1 \cup R^2 \cup \ldots$ → transitive closure of R

~~Def 2 Connectivity relation in P 579~~

ref closure $R \cup R^0$

sym. " $R \cup R^{-1}$

tran. closure $\underbrace{R \cup R^2 \cup R^3 \cup \ldots}$

ref & trans. closure $R^0 \cup R \cup R^2 \cup R^3 \cup \ldots$

ref & sym. "

sym & tran " $R^{-1} \cup R^0 \cup R$

2)

(2

A, $\cancel{X}$ C : set

$R \subseteq A \times A$, $f, g : A \to 2^B$ (setvalued ftn) $\cancel{A \to B}$

($\cancel{g : A \to 2^B}$)

The Art of Programming
— D.E. Knuth

~~Initially~~ $\cancel{\forall a \in A \quad f(a) = \emptyset}$

$$\forall a \in A \quad f(a) \triangleq g(a) \cup \bigcup_{aRb} f(b)$$

$$\Downarrow$$

$$f(a) = \bigcup_{aR^*b} g(b)$$