1)

제 강 Equiv. Rel and Partial Order

9-5 ① ref
② symm.
③ tran.

9-6 ① ref (? ○X)
② antisymmetric
③ ~~(a)~~ trans.

Three faces of rel

Three properties " $R \subseteq A \times A$.

1 Ref: $R \supseteq id_A = R^0$

2. ~~symm~~ irref $\longrightarrow$ $R \cap id_A = \emptyset$ 서로소 (mutually disjoint)

2. symmetric $aRb \Rightarrow bRa$ – 동등 사회 (Ex =

asymmetic $aRb \Rightarrow b \not R a$ 계급 사회 (Ex <, >, $\not\geq$, $\not\leq$ ...)

antisymmetric (수학적 touch) $(aRb) \wedge (bRa) \Rightarrow a = b$ " (예 <, >, ≤, ≥ ... O.K.

asymm. 의 as (장난)

9.4 3. transitive $aRb \wedge bRc \Rightarrow aRc$. $R^n \subseteq R$

Thm 2

Def Closure of {ref, symm, tran}

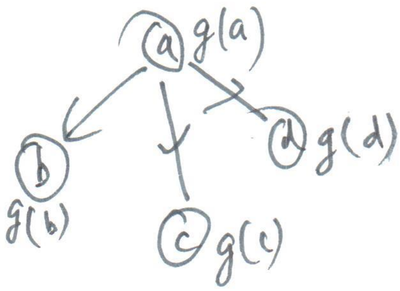
ref. closure of R: $R \cup id_A = R^0$

symm. " of R: $R \cup R^{-1}$

tran. " of R: $R^+ \triangleq R \cup R^2 \cup R^3 \cup \cdots = \bigcup_{i \in N_1} R^i$

ref-and tran. closure of R: $R^* \triangleq R^0 \cup R^1 \cup R^2 \cup \cdots = \bigcup_{i \in N_0} R^i$

Path(a, b)



$f(a) = g(a) \cup g(b) \cup g(c) \cup g(d)$

~~x~~ 즉. aRx

9-5 R is equivalent $R \subseteq A \times A$ 1. ref 2. symm. 3. trans. $(= \subseteq \mathbb{N}\times\mathbb{N})$

$[a]_R = \{ b \in A \mid aRb \}$

equivalent class

$\bigcup_{a\in A} [a]_R = A$ $\quad$ $[a]_R \cap [b]_R = \emptyset$ $\quad a \not R\, b$

exhaustive $\quad$ disjoint

$Par_R(A) = \{ [a]_R \subseteq A \mid a \in A \}$

Ex) $\equiv_4 \subseteq \mathbb{Z}\times\mathbb{Z}$

$(5,1) \in \equiv_4$ $\quad (5,2) \notin \equiv_4$

$\cancel{\mathbb{Z}\times\mathbb{Z}} \subseteq A\times A$

$A\times A$: equ. rel. $\quad Par_{A\times A}(A) = \{A\}$ $\quad |Par_{A\times A}(A)| = 1$

$id_A \subseteq A\times A$ " $\quad Par_{id_A}(A) = \{\{a\} \mid a \in A\}$

$|Par_{id_A}(A)| = |A|$

9-6 Partially ordered set

$(\mathbb{Z}, \leq)$ $\quad (\mathbb{Z}^+, \mid)$ $\quad (2^{\{a,b,c\}}, \subseteq)$ lattice

Hasse diagram